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On a pursuit differential game with integral constraints in $\boldsymbol{R^n}$

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Abstract. In this paper, we study a differential game of pursuit for a system of linear ordinary differential equations in the space R^n. Integral constraints are imposed on the control parameters. A pursuer strategy is constructed that guarantees the completion of pursuit problem for a differential game.

Keywords: pursuer, evader, pursuit differential game, control, admissibility of strategy, completion of pursuit, **.**

Аннотация. В данной работе изучается дифференциальная игра преследования для конечной системы линейных обыкновенных дифференциальных уравнений уравнений в пространстве R^n. На параметры управления наложены интегральные ограничения. Строится стратегия преследователя, гарантирующая завершение задачи преследования для дифференциальной игры.

Ключевые слова: преследователь, убегающий**,** дифференциальная игра преследования, управление, допустимость стратегии, завершение преследования.

Annotatsiya. Ushbu maqolada biz R^n fazoda chiziqli oddiy differensial tenglamalar tizimiga intilishning differentsial o'yinini o'rganamiz. Boshqarish parametrlariga integral cheklovlar qo'yiladi. Differensial o'yin uchun ta'qib qilish muammosini yakunlashni kafolatlaydigan ta'qibchilar strategiyasi tuzilgan.

Kalit so'zlar: ta'qibchi, qochish, ta'qib qilish differensial o'yini, nazorat, strategiyaning maqbulligi, ta'qibni yakunlash, .

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1. The pursuit differential game of one pursuer and one evader

We study the following differential game

$$\begin{aligned} P:&\quad \dot{x} = -ax + u(t), \quad x(0) = x_0, \\ E:&\quad \dot{y} = -bx + v(t), \quad y(0) = y_0, \end{aligned} \tag{1}$$

where $t \geq 0, x, y, x_0, y_0 \in R^n$, $x_0 \neq y_0$, $b \geq a > 0$ and u, v are the controls of the pursuer P and evader E respectively, usually subjected to integral constraints

$$\int_0^\infty |\,u(s)|^2 ds \leq \rho^2, \tag{2}$$

and

$$\int_0^\infty |\,v(s)|^2 ds \leq \sigma^2. \tag{3}$$

The solution of equation (1) is

$$x(t) = x_0 e^{-at} + \int_0^t e^{-a(t-s)}\, u(s) ds,$$

$$y(t) = y_0 e^{-bt} + \int_0^t e^{-b(t-s)}\, v(s) ds.$$

The attainability set of the pursuer P from the initial state x_0 at the time $t = 0$ to the time $t = \theta$ is the ball $H(x_0 e^{-a\theta}, \varphi(a, \theta)\rho)$ of radius $\varphi(a, \theta)\rho$ and centred at $x_0 e^{-a\theta}$, where

$$\varphi(a,\theta)=\left(\int_0^{\theta} e^{-2a(\theta-s)}ds\right)^{\frac{1}{2}}=\sqrt{\frac{1}{2a}(1-e^{-2a\theta})}. \qquad (4)$$

Definition. *In the game (1), the pursuit is said to be completed within the time $\theta>0$, if there exists control $u=u(t,v)$ of pursuer P such that $x(t)=y(t)$ holds for any admissible control $v=v(t)$ of the evader E, for some $t\in[0,\theta]$.*

The main result of this paper is to show that if the control resource of the pursuer is greater than that of the evader, then completion of pursuit is guaranteed.

Theorem 1. *If $\rho>\sigma$, then pursuit can be completed for the finite time $\theta>0$ in the game (1).*

Proof. To prove the theorem we first find a formula for a guaranteed pursuit time. Then we show the pursuit can be completed in the game by choosing a control to pursuer.

1) A formula for a guaranteed pursuit time. Define θ by the equation

$$\varphi(a,\theta)\rho=|x_0e^{-a\theta}-y_0e^{-b\theta}|+\varphi(a,\theta)\sigma.$$

This equation has a root since

$$\text{if } \theta=0,\ 0<|x_0-y_0|+0,$$
$$\text{if } \theta\to\infty,\ \frac{\rho}{\sqrt{2a}}>|0|+\frac{\sigma}{\sqrt{2a}}.$$

Next we claim that the control

$$u_0(s)=\frac{y_0e^{-b\theta}-x_0e^{-a\theta}}{\varphi^2(a,\theta)}e^{-a(\theta-s)}$$

transfers the point $x_0e^{-a\theta}$ to $y_0e^{-b\theta}$. Indeed,

$$\begin{aligned}x(\theta)\ &=x_0e^{-a\theta}+\int_0^{\theta}e^{-a(\theta-s)}u_0(s)ds=x_0e^{-a\theta}+\frac{y_0e^{-b\theta}-x_0e^{-a\theta}}{\varphi^2(a,\theta)}\int_0^{\theta}e^{-2a(\theta-s)}ds=\\ &=x_0e^{-a\theta}+y_0e^{-b\theta}-x_0e^{-a\theta}=y_0e^{-b\theta}.\end{aligned}$$

2) The strategy for the pursuer is

$$\begin{gathered}u(s)=u_0(s)+e^{(a-b)(\theta-s)}v(s),\ \ 0\le s\le\theta,\\ u(s)=0,\ \ s>\theta.\end{gathered} \qquad (5)$$

3) Admissibility of the strategy (5). By (2)-(4), and $b\ge a$, we have

$$\left(\int_0^{\theta}|u(s)|^2ds\right)^{1/2}\le\left(\int_0^{\theta}|u_0(s)|^2ds\right)^{1/2}+\left(\int_0^{\theta}e^{2(a-b)(\theta-s)}|v(s)|^2ds\right)^{1/2}$$

$$\le\frac{|y_0e^{-b\theta}-x_0e^{-a\theta}|}{\varphi^2(a,\theta)}\left(\int_0^{\theta}e^{-2a(\theta-s)}ds\right)^{1/2}+\left(\int_0^{\theta}|v(s)|^2ds\right)^{1/2}\le\rho-\sigma+\sigma=\rho.$$

4) Pursuit is completed at $t=\theta$.

$$
\begin{aligned}
x(\theta) &= x_0 e^{-a\theta} + \int_0^\theta e^{-a(\theta-s)} \left(u_0(s) + e^{(a-b)(\theta-s)} v(s)\right) ds \\
&= x_0 e^{-a\theta} + \int_0^\theta e^{-a(\theta-s)}\, u_0(s) ds + \int_0^\theta e^{-a(\theta-s)}\, e^{(a-b)(\theta-s)} v(s) ds \\
&= x_0 e^{-a\theta} + \frac{y_0 e^{-b\theta} - x_0 e^{-a\theta}}{\varphi^2(a,\theta)} \int_0^\theta e^{-2a(\theta-s)}\, ds + \int_0^\theta e^{-b(\theta-s)}\, v(s) ds \\
&= x_0 e^{-a\theta} + y_0 e^{-b\theta} - x_0 e^{-a\theta} + \int_0^\theta e^{-b(\theta-s)}\, v(s) ds = y(\theta).
\end{aligned}
$$

This completes the proof.

2. The pursuit differential game of many pursuers and one evader

Now, we study the next case which has many pursuers $P_i,\ i = 1,2,\dots,m$, and one evader whose motions described by the equations

$$
\begin{aligned}
&P_i\colon\ \dot{x} = -ax + u_i(t), \quad x_i(0) = x_{i0}, \\
&E\colon\ \dot{y} = -bx + v(t), \quad y(0) = y_0,
\end{aligned}
\tag{6}
$$

where $t \ge 0$, $x_i, y, x_{i0}, y_0 \in R^n$, $x_{i0} \ne y_0$, $b > \frac{a_i}{2} > 0$ and u_i, v for $i = 1,\dots,m$, are the controls of the i-th pursuer P_i and evader E respectively, usually subjected to integral constraints

$$
\int_0^\infty |\, u_i(s)|^2 ds \le \rho_i^2, \tag{7}
$$

and

$$
\int_0^\infty |\, v(s)|^2 ds \le \sigma^2. \tag{8}
$$

We say that pursuit can be completed in the game within the time $T > 0$, if there exists admissible strategies $u_1 = u_1(t,v), u_2 = u_2(t,v),\dots,u_m = u_m(t,v)$, of pursuers such that $x_i(t) = y(t)$ holds at some $i = \{1,\dots,m\}$, for any admissible control $v = v(t)$ of the evader for some $t \in [0,T]$.

Now, we show that if the total control resources of the pursuers is greater than that of the evader, then completion of pursuit is guaranteed.

Theorem 2. *If $\sum_{i=1}^m \rho_i^2 > \sigma^2$, then pursuit can be completed for the finite time $\theta > 0$ in the game (6).*

Proof. Let $\rho = (\rho_1^2 + \rho_2^2 + \dots + \rho_m^2)^{1/2}$, $\sigma_i = \frac{\rho_i \sigma}{\rho}$ and

$$
F_i(\tau,t) = \frac{|x_i(\tau)e^{-at} - y(\tau)e^{-bt}|^2}{\varphi^2(a,\tau,t)},\ t > \tau,\ i = 1,2,\dots,m, \tag{9}
$$

where

$$
\varphi(a,\tau,t) = \left(\int_\tau^t e^{-2a(t-s)} ds\right)^{\frac{1}{2}} = \sqrt{\frac{1}{2a}(1 - e^{-2a(t-\tau)})}. \tag{10}
$$

Since $\frac{1}{\varphi(a,\tau,t)} \to +\infty$ as $t \to \tau^+$ for a fixed $\tau \ge 0$, the left part of the equation

$$
F_i^{\frac{1}{2}}(\tau,t) = \rho_i - \sigma_i \tag{11}
$$

approaches $+\infty$ as $t \to \tau^+$. Moreover, the function (9) is decreasing function of t and $F_i(\tau, t) \to 0$ as $t \to +\infty$. Indeed, by $b \geq a$ we have

$$F_i(\tau,t) = \frac{\| x_i(\tau)e^{-at} - y(\tau)e^{-bt} \|^2}{\varphi^2(a,\tau,t)} \leq \frac{|x_i(\tau)e^{-at} - y(\tau)e^{-at}|^2}{\frac{1}{2a}(1-e^{-2a(t-\tau)})} =$$
$$= \frac{2a|x_i(\tau)-y(\tau)|^2}{e^{2at}(1-e^{-2a(t-\tau)})} = \frac{2a|x_i(\tau)-y(\tau)|^2}{e^{2at}-e^{2a\tau}} \to 0, \qquad t \to \infty.$$

Thus, (11) has a unique solution $t = \theta_i$.

Construction of the pursuers strategies. Our recommendation is for the pursuers to act sequentially and use parallel approach control to capture the evader. In doing so, they can utilize their extra control resources to deplete the evader's resources at a predetermined time. This is based on the assumption that the evader will use a certain amount of their control resource. If the evader uses more than anticipated, the pursuer will use up all their resources sooner than planned. However, if the evader uses less, the pursuer will be able to catch them without needing additional pursuers.

The strategy of pursuers $P_i,\ i = 1,2,\dots,m$ as follows. Set

$$u_1(s,y(s),v(s)) = \begin{cases} \dfrac{y_0e^{-bt_1} - x_0e^{-at_1}}{\varphi^2(a,0,t_1)} e^{-a(t_1-s)} + e^{(b-a)(t_1-s)}v(s), \text{if } 0 \leq s \leq t_1, \\ 0, \qquad \text{if } s > t_1. \end{cases}$$

$$u_i\big(s,y(s),v(s)\big) = 0,\ 0 \leq s \leq t_1,\ i = 2,3,\dots,m, \tag{12}$$

where time

$$t_1 = \begin{cases} \theta_1, \text{if } \tau_1 \geq \theta_1, \\ \tau_1,\ \ \text{if } \tau_1 < \theta_1, \end{cases}$$

and τ_1 is the first time when $\int_0^{\tau_1} |\, v(s)|^2 ds = \sigma_1^2$ which may or may not exists.

Consequently,

$$\sigma(\tau_1) = \sigma^2 - \int_0^{\tau_1} |\, v(s)|^2 ds = \sigma^2 - \sigma_1^2.$$

If such time τ_1 fails to exists, then we set $\tau_1 = +\infty$.

The case $\tau_1 = +\infty$ yields the inequality $\sigma(t) > \sigma^2 - \sigma_1^2$, for all $t \geq 0$ which follows from

$$\sigma(\tau_1) = \sigma^2 - \int_0^{t} |\, v(s)|^2 ds > \sigma^2 - \int_0^{\infty} |\, v(s)|^2 ds = \sigma^2 - \sigma_1^2,\ \ t \geq 0.$$

Hence, we have

$$\int_0^{\tau_1} |\, v(s)|^2 ds = \int_0^{\infty} |\, v(s)|^2 ds \leq \sigma_1^2.$$

But if $\tau_1 \geq \theta_1$, that is, $t_1 = \theta_1$, then we claim pursuit can be completed at time t_1. Observe that, for this case,

$$\int_0^{\theta_1} |\, v(s)|^2 ds = \int_0^{\tau_1} |\, v(s)|^2 ds \leq \sigma_1^2.$$

Completion of pursuit. To show completion of pursuit, we first establish the admissibility the pursuers strategy (12). By (9)-(11) and $b \geq a$, we have

$$\left(\int_0^{t_1} |u_1(s)|^2 ds\right)^{1/2} = \left(\int_0^{\theta_1} |u_1(s)|^2 ds\right)^{1/2} \leq \left(\int_0^{t_1} \frac{\left|y_0 e^{-b\theta_1} - x_0 e^{-a\theta_1}\right|^2}{\varphi^4(a,0,\theta_1)} e^{-2a(\theta_1 - s)} ds\right)^{1/2} +$$

$$+\left(\int_0^{\theta_1} e^{2(a-b)(\theta_1-s)} |v(s)|^2 ds\right)^{1/2} \leq \frac{\left|y_0 e^{-b\theta_1} - x_0 e^{-a\theta_1}\right|}{\varphi^2(a,0,\theta_1)} \left(\int_0^{\theta_1} e^{-2a(\theta_1-s)} ds\right)^{1/2} + \left(\int_0^{\theta_1} |v(s)|^2 ds\right)^{1/2}$$

$$\leq F_1^{1/2}(0,\theta_1) + \sigma_1 = \rho_1 - \sigma_1 + \sigma_1 = \rho_1.$$

Hence, we conclude that the strategy (12) is admissible for the case $\tau_1 \geq \theta_1$. And if the pursuers adopt the strategy (12), then we have

$$\begin{aligned} x(t_1) &= x_0 e^{-a\theta_1} + \int_0^{\theta_1} e^{-a(\theta_1-s)} \left(\frac{y_0 e^{-b\theta_1} - x_0 e^{-a\theta_1}}{\varphi^2(a,0,\theta_1)} e^{-a(\theta_1-s)} + e^{(b-a)(\theta_1-s)} v(s)\right) ds \\ &= x_0 e^{-a\theta_1} + \int_0^{\theta_1} e^{-a(\theta_1-s)} \frac{y_0 e^{-b\theta_1} - x_0 e^{-a\theta_1}}{\varphi^2(a,0,\theta_1)} e^{-a(\theta_1-s)} ds + \int_0^{\theta_1} e^{-a(\theta_1-s)}\, e^{(a-b)(\theta_1-s)} v(s) ds \\ &= x_0 e^{-a\theta_1} + \frac{y_0 e^{-b\theta_1} - x_0 e^{-a\theta_1}}{\varphi^2(a,0,\theta_1)} \int_0^{\theta_1} e^{-2a(\theta_1-s)}\, ds + \int_0^{\theta_1} e^{-b(\theta_1-s)}\, v(s) ds \\ &= x_0 e^{-a\theta_1} + y_0 e^{-b\theta_1} - x_0 e^{-a\theta_1} + \int_0^{\theta_1} e^{-b(\theta_1-s)}\, v(s) ds = y(\theta_1) = y(t_1). \end{aligned}$$

This implies that pursuit can be completed in the game (6) at time $t_1 = \theta_1$. Thus, if the pursuers apply the admissible strategy (12), then either the evader spend the resource of control less than or equal to σ_1^2 or pursuit will be completed.

If we assume that pursuit fails to be completed on the time interval $[0, t_1]$, then we must have $\tau_1 < \theta_1$ which implies $[0, t] = [0, \tau_1]$.

Using mathematical induction, we define the numbers θ_i and τ_i for $i = 1,2,\ldots,m$ as follows.

Let the numbers $\theta_1, \tau_1, \theta_2, \tau_2, \ldots, \theta_i, \tau_i,\ (i = 1,2,\ldots)$ be defined subject to the following conditions:

- **i.** $F_i(\tau_{i-1}, t)$, where $\tau_0 = 0$ has a unique solution at $t = \theta_i$;
- **ii.** τ_i is the first instant when $\int_{\tau_{i-1}}^{\tau_i} |\, v(s)|^2 ds = \sigma_i^2$, for all $i = 1,2,\ldots,m$. That is, $\sigma(\tau_i) = \sigma^2 - \sum_{j=1}^{i} \sigma_j^2$, such time may exist or not. For the latter case, we let $\tau_i = +\infty$;
- **iii.** $\tau_k < \theta_k$, $k = 1,2,\ldots,i$, and pursuit fails to be completed on $[0, \tau_i]$. In particular, $\tau_k < \infty$, $k = 1,2,\ldots,i$.

We now define $t = \theta_{i+1}$ as a unique solution of the equation

$$F_{i+1}^{1/2}(\tau_i, t) = \rho_{i+1} - \sigma_{i+1},$$

where

$$F_{i+1}(\tau_i, t) = \frac{|x_{i+1}(\tau_i) e^{-at} - y(\tau_i) e^{-bt}|^2}{\varphi^2(a, \tau_i, t)}.$$

The strategy of pursuersfor all $t \geq \tau_i$, is defined as follows:

Set

$$u_{i+1}(s,y(s),v(s))=\begin{cases}0, & \text{if } s<\tau_i\\ \dfrac{y(\tau_i)e^{-bt_{i+1}}-x(\tau_i)e^{-at_{i+1}}}{\varphi^2(a,\tau_i,t_{i+1})}e^{-a(t_{i+1}-s)}+e^{(b-a)(t_{i+1}-s)}v(s), & \text{if } \tau_i\le s\le t_{i+1},\\ 0, & \text{if } s>t_{i+1},\end{cases}$$

$$u_j\big(s,y(s),v(s)\big)=0,\ \tau_i\le s\le t_{i+1},\ j=1,2,\dots,i,i+2,\dots,m, \tag{13}$$

where time

$$t_{i+1}=\begin{cases}\theta_{i+1}, \text{if } \tau_{i+1}\ge\theta_{i+1},\\ \tau_{i+1}, \ \text{if } \tau_{i+1}<\theta_{i+1},\end{cases}$$

and τ_{i+1} is the first time when $\int_{\tau_i}^{\tau_{i+1}}|\,v(s)|^2ds=\sigma_{i+1}^2$ which may or may not exists.

Consequently,

$$\sigma(\tau_{i+1})=\sigma^2-\int_{\tau_0}^{\tau_{i+1}}|\,v(s)|^2ds=\sigma^2-(\sigma_1^2+\dots+\sigma_{i+1}^2).$$

If τ_{i+1} fails to exists, then we set $\tau_{i+1}=+\infty$, which yields

$$\int_{\tau_i}^{\tau_{i+1}}|\,v(s)|^2ds=\sigma(\tau_i)-\sigma(\tau_{i+1})\le\sigma_{i+1}^2.$$

For the case $\tau_{i+1}\ge+\theta_{i+1}$, observe that

$$\int_{\tau_i}^{\theta_{i+1}}|\,v(s)|^2ds\le\int_{\tau_i}^{\tau_{i+1}}|\,v(s)|^2ds\le\sigma_{i+1}^2.$$

Admissibility of the strategy (13) is shown by using similar argument in the admissibility of (12). That is,

$$\left(\int_{\tau_i}^{\theta_{i+1}}|u_{i+1}(s)|^2ds\right)^{1/2}\le\left(\int_{\tau_i}^{\theta_{i+1}}\frac{\left|y(\tau_i)e^{-b\theta_{i+1}}-x(\tau_i)e^{-a\theta_{i+1}}\right|^2}{\varphi^4(a,\tau_i,\theta_{i+1})}e^{-2a(\theta_{i+1}-s)}ds\right)^{1/2}$$

$$+\left(\int_{\tau_i}^{\theta_{i+1}}e^{2(a-b)(\theta_{i+1}-s)}|v(s)|^2ds\right)^{1/2}\le\frac{\left|y(\tau_i)e^{-b\theta_{i+1}}-x(\tau)_ie^{-a\theta_{i+1}}\right|}{\varphi^2(a,\tau_i,\theta_{i+1})}\left(\int_{\tau_i}^{\theta_{i+1}}e^{-2a(\theta_{i+1}-s)}ds\right)^{1/2}$$

$$+\left(\int_{\tau_i}^{\theta_{i+1}}|v(s)|^2ds\right)^{1/2}\le F_{i+1}^{1/2}(\tau_i,\theta_{i+1})+\sigma_{i+1}=\rho_{i+1}-\sigma_{i+1}+\sigma_{i+1}=\rho_{i+1}.$$

In addition, the strategy (13) ensures $x(\theta_{i+1})=y(\theta_{i+1})$ if adopted by the pursuers on the time interval $[\tau_i,\tau_{i+1}]$. That is, pursuit will be completed at time θ_{i+1} and so on. And if pursuit goes until time τ_{m_1} without completion, then let θ_m be the unique solution of

$$F_m^{1/2}(\tau_{m-1},t)=\rho_m-\sigma_m,$$

where

$$F_m(\tau_{m-1},t)=\frac{|x_m(\tau_{m-1})e^{-at}-y(\tau_{m-1})e^{-bt}|^2}{\varphi^2(a,\tau_{m-1},t)}.$$

Since we have $\sigma(\tau_{m-1}) = \sigma^2 - (\sigma_1^2 + \cdots + \sigma_{m-1}^2) = \sigma_m^2$, then

$$\int_{\tau_{m-1}}^{\theta_m} |\, v(s)|^2 ds = \sigma(\tau_{m-1}) - \sigma(\tau_m) \le \sigma(\tau_{m-1}) = \sigma_m^2.$$

Let

$$u_m\big(s, y(s), v(s)\big) = \begin{cases} 0, & \text{if } s < \tau_{m-1} \\ \dfrac{y(\tau_{m-1})e^{-bt_m} - x(\tau_{m-1})e^{-at_m}}{\varphi^2(a, \tau_{m-1}, t_m)} e^{-a(t_m - s)} + e^{(b-a)(t_m - s)} v(s), \\ 0, & \text{if } s > t_m, \end{cases}$$

$$\text{if}\tau_{m-1} \le s \le t_m,\ u_j(s, y(s), v(s)) = 0,\ \ s > \tau_{m-1},\ j = 1,2,\dots,i,i+2,\dots,m-1. \quad (14)$$

Admissibility of the strategy (14) is shown by using similar argument in (12) again. Thus,

$$\left(\int_{\tau_{m-1}}^{\theta_m} |u_m(s)|^2 ds \right)^{1/2} \le \rho_m.$$

It can also be verified that the strategy (14) ensures $x(\theta_m) = y(\theta_m)$ if adopted by the pursuers on the time interval $[\tau_{m-1}, \tau_m]$. That is, pursuit is completed at time θ_m. This finishes the proof.

$\boldsymbol{R^n}$ fazoda integral chegaralanishli ta'qib qilishning differensial o'yini haqida

Kalit so'zlar: ta'qib qiluvchi, qochuvchi, ta'qib qilish differensial o'yini, boshqaruv, boshqaruvning joizligi, ta'qib qilishning yakunlanishi.

Ushbu maqolada, R^n fazoda dastlab, harakat tenglamalari (1) ko'rinishda berilgan ta'qib qiluvchi va qochuvchining differensial o'yinini o'rgandik. 1-teorema orqali ta'qibchining boshqaruv resursi qochuvchining boshqaruv resursidan katta bo'lganda ya'ni (2) va (3) joizlik shartlari uchun $\rho > \sigma$ tengsizlik bajarilganda ta'qib qilishni tugatish mumkinligini ko'rsatdik. Bunda ta'qibchi (5) boshqaruv bilan harakat qilganda har qanday vaqt va qochuvchining har qanday boshqaruvi uchun (2) joizlik sharti bajarilishini tekshirdik. So'ngra, tanlangan boshqaruv yordamida ta'qib qilishni chekli vaqtda tugatish mumkinligini ko'rsatdik.

Ishning davomida harakat tenglamalari (6) ko'rinishda berilgan m ta ta'qib qiluvchi va bir qochuvchining differensial o'yinini o'rgandik. Bu holda asosiy natija sifatida 2-teoremani keltirdik, ya'ni taq'ibchilarning energiya resurlarining yig'indisi qochuvchining energiya resursidan katta bo'lganda ta'qib qilishni tugatish mumkinligini ko'rsatdik. Biz ta'qibchilarga navbatma-navbat harakat qilishni va ta'qibni tugatish uchun parallel yaqinlashish boshqaruvini qo'llashni taklif qildik. Agar qochuvchi o'z boshqaruv resursining belgilangan qismini sarf qilsa, ta'qibchilar o'zlarining ortiqcha nazorat resursidan barcha resurslarni oldindan hisoblab chiqilgan vaqtda sarflash uchun foydalanadilar. Agar qochuvchi rejalashtirilganidan ko'proq resurs sarflasa, ta'qib qiluvchi barcha boshqaruv resurslarini kutilganidan oldinroq ishlatadi. Agar qochuvchi kamroq resurs sarflasa, mos o'rindagi ta'qib qiluvchi keyingi ta'qibchilar jalb qilinishidan oldin uni ushlaydi. Agar qochuvchi $m-1$ ta ta'qibchidan qochib ketishni uddalagan taqdirda ham, urning ortib qolgan boshqaruv resursi oxirgi ta'qibchining boshqaruv resursidan kichikligi hisobiga va 1-teoremaga ko'ra ta'qib qilish yakunlanadi.

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