ЗАХИРИДДИН
МУҲАММАД БОБУР
НОМИДАГИ АНДИЖОН
ДАВЛАТ УНИВЕРСИТЕТИ

ANDIJAN STATE
UNIVERSITY NAMED
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ИЛМИЙ ХАБАРНОМА

Физика-математика
Тадқиқотлари
(Махсус сон)

SCIENTIFIC BULLETIN

Physical and
Mathematical Research
(Special Issue)

Андижон
2023 йил

Муассис

Захириддин Мухаммад Бобур номидаги Андижон давлат университети

ИЛМИЙ ХАБАРНОМА.
ФИЗИКА-МАТЕМАТИКА
ТАДҚИҚОТЛАРИ

Журнал бир йилда 2 марта чоп этилади.

Андижон вилояти ахборот ва оммавий коммуникациялар бошқармаси
томонидан 2019-йил 26 декабрда
0452 рақам билан рўйхатга олинган.

Нашр индекси: 344

Нашр учун масъул:
А.Й.Бобоев

Босишга рухсат этилди:
27.12.2019.
Қоғоз бичими: 60x81 1/8
Босма табоғи: 13,5
Офсет босма. Офсет қоғози.
Адади: 110 дона.
Баҳоси келишилган нархда.
Буюртьа №: 165.
“Муҳаррир” нашриёти манбаа бўлимида чоп этилди.
Тошкент шаҳри, Сўгалли ота кўчаси 7-уй

Таҳририят манзили:

170100, Андижон шаҳри, Университет кўчаси, 129. Телефон: +998911602043.
Факс: (374) 223-88-30

E-mail: adu_xabarnoma@mail.ru Расмий сайт: uzjournals.edu.uz/adu

Сборник статей международной научно-практической конференции по «Полупроводниковая опто- и наноэлектроника, альтернативные источники энергии и их перспективы» Андижан, 12-13 октября 2023 года

ОГЛАВЛЕНИЕ

Application of Solution of the Quantum Kinetic Equations for Reneweble Energy problem

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Abstract. This article considers a non-equilibrium system of electrons interacting with an electromagnoite field. For this system formule intensity of emittting of both the electromagnetic field emitting from the system of electrons and its absorption is received. The conditions for the coefficients in the operators of spins w_0, electromagnetic field w_k, interaction between electrons and the field g_k and

$$N_k = \frac{e^{-\beta\hbar\omega(k)}}{1-e^{-\beta\hbar\omega(k)}}$$

are also determined, under which the interaction process occurs linearly and non-linearly. These studies can be used to select efficient materials for converters of solar energy into electricity and in operate the processes.

Introduction. In 1872, for the first time, to describe the evolution of a classical particle, Ludwig Boltzmann introduced a kinetic equation [1] for the distribution function depending on the coordinate and momentum of the particles, called the following name. The other most well-known kinetic equation describing plasma evolution is the Vlasov equation [2]. In 1902, Gibbs introduced [3] the equation into statistical mechanics to describe the evolution of many interacting particles, which was derived as early as 1838 by Liouville. In 1946, starting from the Liouville kinetic equation, a chain of kinetic equations was formulated that relates the Liouville equation [4] and the Boltzmann equation and the Vlasov equation. This chain was included in the physical literature as the Bogolyubov-Born-GreenKirkwood-Yvon (BBGKY) chain [5], since it summarizes the attempts of different authors to generalize the kinetic equations for a single particle for the case of systems of many interacting particles. In quantum mechanics, particle dynamics is described by the Schro¨dinger equation [6]. To describe the physical phenomena in semiconductor physics, in solidstate physics, the Hartley equation [7] and other equations are used. To describe the evolution of systems of many interacting particles, the Liouville quantum kinetic equation is also used. Similarly, the classical case for connecting the equations for the one-particle case and the equation for the case of many particles, proceeding from the Liouville quantum equation, have been derived the BBGKY chain for the quantum kinetic equations [8], [9] for the density matrices. As is known, the solution of the classical and quantum chain of BBGKY allows to determine the distribution function and the density matrix, respectively, in the classical and quantum cases. The definition of these solutions allows using these results to calculate the average values of physical quantities, characterizing the considered system of particles. In all areas of physics, the above equations are used to describe the dynamics of the corresponding particle systems. In this paper, on the basis of the Liouville quantum kinetic equation we show the possibility of using them for studying problem of renewable energy.

Application of the quantum kinetic equations for the solution of problems of reneweble energy.the paper, Jaynes-Cummings Model (JCM) [10] is investigated in terms of the methods [11]-[16]. The system under consideration includes *N* two-level atoms, interacting with electromagnetic field mode. Hamiltonian of such systems in notations [14, 15] is given by: In the paper, Jaynes-Cummings Model (JCM) [10] is investigated interms of the methods [11]-[16]. The system under consideration includes *N* two-level atoms, interacting with electromagnetic field mode. Hamiltonian of such systems in notations [14, 15] is given by:

$$\hat{H}_t = \hat{H}_0 + \sum_{j=1}^{N} \hbar\omega_0 \hat{S}_j^z + \sum_{k=1}^{N} \hbar\omega_k \hat{b}_k^\dagger \hat{b}_k +$$

$$+e^{\epsilon t} \sum_{k,j=1}^{N} \frac{\hbar g_k}{\sqrt{N}} (e^{ikx_j} \hat{b}_k \hat{S}_j^-(\mu) + e^{-ikx_j} \hat{b}_k^\dagger \hat{S}_j^+(\mu)), \qquad (1)$$

where the first term describes the energy of free atom. This is given by:

$$\hat{H}(S) = \hat{H}_0 + \sum_{j=1}^{N} \hbar\omega_0 \hat{S}_j^z.$$

Here $\hat{H}_0$ is the kinetic energy of atoms.
The second term corresponds to free electromagnetic field and is given by:

$$\hat{H}(\Sigma) = \sum_{k=1}^{N} \hbar\omega_k \hat{b}_k^\dagger \hat{b}_k,$$

where $\hat{b}_k$ and $\hat{b}_k^\dagger$ are the operators of the annihilations and creations of photon with wave vector k . Here (S) and (Σ) denote (atom) and (field), correspondingly.
The last term corresponds to interaction of atoms with the field and is given by:

$$\hat{H}_t(S,\Sigma) = e^{\epsilon t} \sum_{k,j=1}^{N} \frac{\hbar g_k}{\sqrt{N}} (e^{ikx_j} \hat{b}_k \hat{S}_j^-(\mu) + e^{-ikx_j} \hat{b}_k^\dagger \hat{S}_j^+(\mu)),$$

where g_k is the dipole coupling strength, N is the number of atoms, $\hat{S}_j^-(\mu) = \hat{S}_j^+ + \mu\hat{S}_j^-$; $\varepsilon, \mu \in \mathcal{R}$; x_j is the radius vector of the j - th atom and $0 \leq t$ - time. In (3) $\hat{b}$ and $\hat{b}^\dagger$ are photon annihilation and creation operators, respectively, ω_0 is the splitting frequency between the atomic levels, $\hbar$ is Plank's constant, ω is the frequency of the field mode, g is the dipole coupling strength, and $\hat{S}^+, \hat{S}^-, \hat{S}^z$ are atomic spin operators satisfying the following commutation relation

$$[\hat{S}^\pm, \hat{S}^z] = \mp\hat{S}^\pm, \quad [\hat{S}^+, \hat{S}^-] = 2\hat{S}^z,$$

where $\hat{S}^+ = \frac{1}{2}(\hat{\sigma}_x + i\hat{\sigma}_y)$, $\hat{S}^- = \frac{1}{2}(\hat{\sigma}_x - i\hat{\sigma}_y)$, $\hat{S}^z = \frac{1}{2}\hat{\sigma}_z$; $\{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$ are Pauli's matrices.
By using the method of elimination of boson variables developed in [11], Liouville equation for operator $f(S)$ in Heisenberg representation can be converted to the following form

$$\mathrm{Tr}_{(S)} \left(f(S) \frac{\partial \rho_t(S)}{\partial t} + \frac{\hat{H}(S) f(S) - f(S) \hat{H}(S)}{i\hbar} \rho_t(S) \right) =$$

$$= \sum_{k,j=1}^{N} \frac{g_k^2}{N} \int_{t_0}^{t} d\tau \mathrm{Tr}_{(S,\Sigma)} e^{-i\omega_k(t-\tau)} e^{\epsilon(t+\tau)} \{ N_k e^{ikx_j} \hat{S}_j^{+}(\tau,\mu) \times$$
$$\times [f(S_t), e^{-ikx_j} \hat{S}_j^{-}(t,\mu)] + (1+N_k)[e^{-ikx_j} \hat{S}_j^{-}(t,\mu), f(S_t)] \times$$
$$\times e^{ikx_j} \hat{S}_j^{+}(\tau,\mu)\} D_{t_0}(S,\Sigma) + \sum_{k,j=1}^{N} \frac{g_k^2}{N} \int_{t_0}^{t} d\tau \mathrm{Tr}_{(S,\Sigma)} e^{i\omega_k(t-\tau)} e^{\epsilon(t+\tau)} \times$$
$$\times \{(1+N_k) e^{-ikx_j} \hat{S}_j^{-}(\tau,\mu)[f(S_t), e^{ikx_j} \hat{S}_j^{+}(t,\mu)] +$$
$$+ N_k [e^{ikx_j} \hat{S}_j^{+}(t,\mu), f(S_t)] e^{-ikx_j} \hat{S}_j^{-}(\tau,\mu)\} D_{t_0}(S,\Sigma),$$

where $N_k = \frac{e^{-\beta\hbar\omega(k)}}{1-e^{-\beta\hbar\omega(k)}}$, and $f(S_t)$ is the dynamic value, $\rho(S)$ is the reduced density matrix of S system, $D(S,\Sigma)$ is the statistical operator of (S,Σ) system.

In paper we used statistical approach and consider JCM from the point of view of many particle systems, Thereby, we used collective operators and took into account the inhomogeneous Lorentz broadening and received formule intensity of emittting $(I(t) > 0)$:

$$I_{\mathrm{emit}}(t) = \frac{\hbar\omega_0\alpha}{4N}(N + \frac{\gamma}{\alpha})^2 sech^2 \frac{\eta(t-t_0)}{2},$$

and under the condition of $\langle \hat{S}^z(t) \rangle < -\frac{N}{2}$ and for $t - t_0$, intensity of absorption $(I(t) < 0)$ rate has the form:

$$I_{\mathrm{abs}}(t) = -\frac{{}^2\omega_0\alpha}{4N}(N + \frac{\gamma}{\alpha})^2 cosech^2 \frac{\eta(t-t_0)}{2}.$$

These latter formulas can be used to select efficient materials for converters of solar energy into electricity.

For the given system we derive the generalized kinetic equation (GKE). For averages $< \hat{\hat{S}}_p^{+}(t) \hat{\hat{S}}_p^{-}(t) >$, under assumption that the interaction between the medium and the field is weak and taking into account a Lorentzian inhomogeneous broadening, we considered the solvability of this GKE equation; additional conditions for g_k, w_k, w_0, N_k were given under which its solution is reduced to the solution of a linear equation, and of the one-dimensional nonlinear Schrödinger equation, respectively.

1.In the case when the relation between g_k, N_k, ω_k and ω_0 remains constant, not depending on k, i.e. the case when $C = \frac{\hbar g_k^2(1+2N_k)}{\omega_0 - \omega_k}$ for any k, where constant value $C > 0$, (for instance the case when $\omega_0 = \omega_k + \hbar g_k^2(1+2N_k)$, $\quad C = 1$) the solution of GKE (16) for $< \hat{S}_p^{+}(t) \hat{S}_p^{-}(t) >$ is reduced to solution of a linear equation.

2.In the case when the relation between N_k, ω_k and ω_0 is $C = \frac{\hbar(1+2N_k)}{\omega_0-\omega_k}$ for any k, where constant value $C > 0$, (for instance the case when $\omega_0 = \omega_k + \hbar(1+2N_k)$, $C = 1$) and g_k^2 is a function which satisfies the condition $F^{-1}\{\psi_p(t)\sum_{k=1}^{N} g_k^2|\psi_k(t)|^2\} = \psi_x(t)|\psi_x(t)|^2$ the solution of GKE is reduced to solution of the one-dimensional nonlinear Schrödinger equation.
This result can be used to understand nonlinear quantum processes occurring in quantum devices.

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